Part IV: More Simple Problem Solving

This part of the tutorial returns to problems that are solved through internal search. You will build Soar programs for a classic AI problem: Missionaries and Cannibals. Other classic AI problems, including Blocks World, the Eight Puzzle, and Towers of Hanoi are included in the set of demonstration programs that comes with the Soar release. This is very similar to the water jug problem you did initially, but has a few interesting extensions. You will start by building the operators, state descriptions, and goal tests that are required to define each problem. You will also be introduced to more of the theory of problem solving based on search in problem spaces. In Part V, you will learn how to modify and extend programs so that they use planning and learning to solve problems.

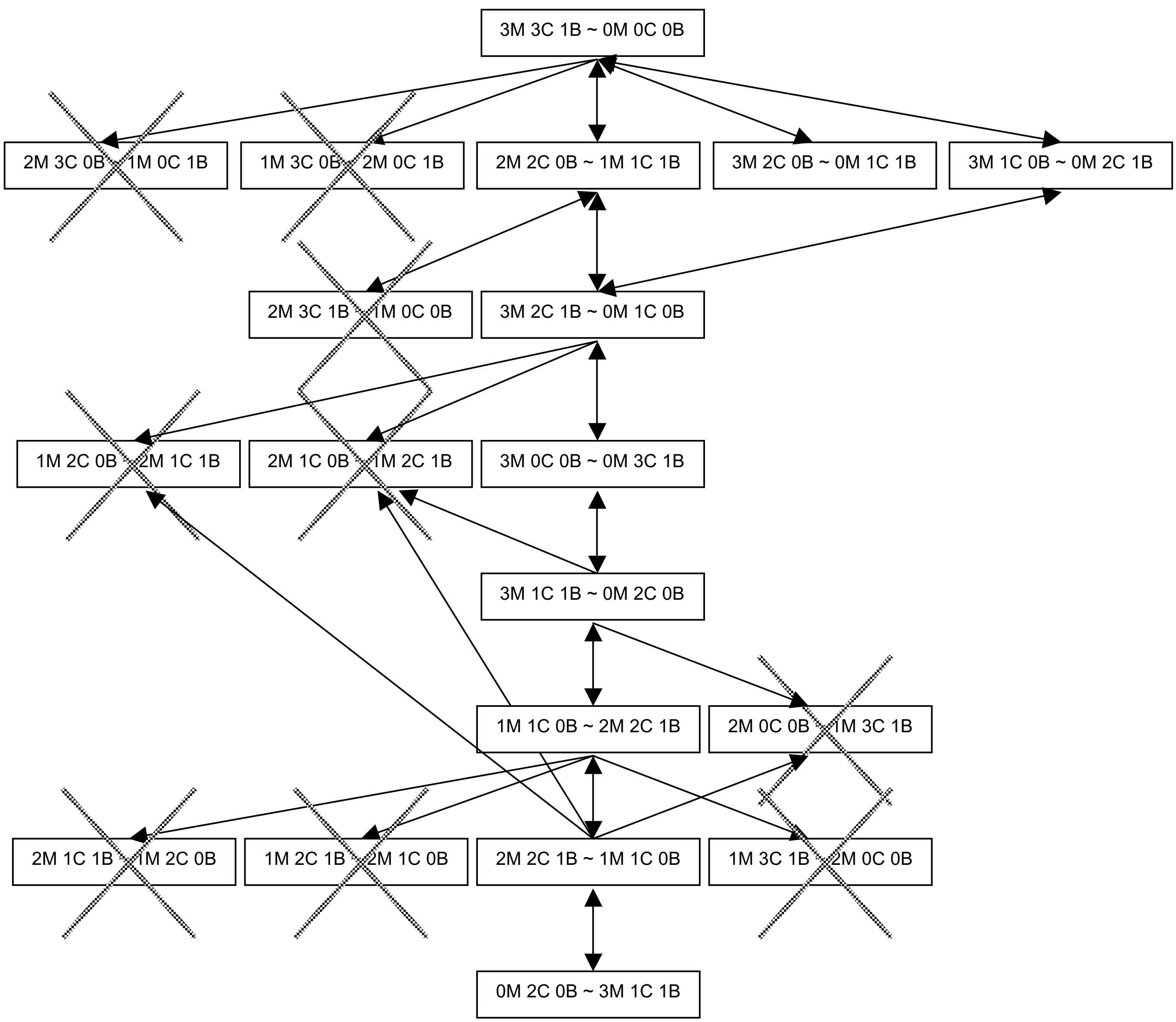
This problem is challenging in a different way than playing games such as Eaters and TankSoar. Eaters and TankSoar are competitive games and they require fast intelligent responses to the current situation, which can change quickly. This problem does not have dynamic environments. However, solving this problem requires selecting the one appropriate operator from a set of many at each decision point. Selecting the correct operator is not easy given the knowledge available from the problem description. The problem can be solved only through trial and error, which involves searching through the space of possible states.

1. Missionaries and Cannibals Definition

Problem Statement:

> *Three missionaries and three cannibals come to a river. There is a boat on their bank of the river that can be used by either one or two persons at a time. This boat must be used to cross the river in such a way that cannibals never outnumber missionaries on either bank of the river (although cannibals can be alone on one bank). How do the missionaries and cannibals successfully cross the river?*

Once again, the first step to creating a Soar program to solve this problem is to decompose it into the problem space (state representation and operators) and the problem (initial state and desired state). One interesting aspect of this problem is that it also includes failure states. If the cannibals ever outnumber the missionaries, then you have failed. Below is a partial graph (it doesn't show the last four steps) of the problem space, which shows that there are many illegal states that need to be avoided along the way to a solution.



Below is a list of the aspects of the problem space and problem that you will define for this problem:

1. The state representation. For this problem this will include the positions of the missionaries, cannibals, and boat, relative to the river.
2. The initial state creation rule. In this problem, all the missionaries, cannibals and the boat are on one bank of the river.
3. The operator proposal rules. For this problem the operators move up to two of the missionaries and/or cannibals across the river with the boat.
4. The operator application rules.
5. The operator and state monitoring rules.
6. The goal recognition rule. In this problem, the desired state is achieved when all missionaries and cannibals have crossed the river.
7. The failure recognition rule. These are rules that detect when a state is created in which the goal cannot be achieved. In this problem, the failure states are whenever the cannibals outnumber the missionaries on one bank of the river.
8. The search control rules.

It may be tempting to try to incorporate the avoidance of failure states into the operators, so that operators are never proposed that lead to failure states. However that is moving an aspect of the problem into the problem space and requires some problem solving to determine what the conditions of the proposal should be. We will see in Part V how Soar can learn to rules that avoid proposing operators when they will lead to failure.

As in the Water Jug problem, this program does not create a plan to solve the problem. Instead, when the program finishes, all of the missionaries and cannibals will have been moved across the river.

2. State Representation

What are the parts of the problem that must be represented on the state? Everything in the problem description is important (there are no irrelevant objects are characteristics of the objects), so an initial list of objects includes three missionaries, three cannibals, a river, and the boat.

At any point in solving the problem, it is necessary to represent which bank of the river the boat is and the banks that the missionaries and cannibals are on. One important observation is that it is not necessary to keep track of each missionary and cannibal individually. All that is important is the *number* of missionaries and cannibals on each bank of the river. For the purposes of this problem all cannibals are the same and all missionaries are the same. Therefore, it is not necessary to have a separate representation of each missionary or cannibal and its current position. Another observation is that you never have to represent a state where the boat has missionaries and cannibals in it – that happens only during the application of an operator. The only states that need to be represented are those with the boat on one bank of the river or the other. Therefore the important aspects of the states that need to be represented are:

- The number of missionaries on each bank of the river.
- The number of cannibals on each bank of the river.
- The bank of the river that the boat is on.

There are many possible ways to represent this information using Soar's attributes and values. Try to come up with one on your own before looking at the representations listed below.

In creating the representations below, the two banks of the river are named left and right, with left being the bank of the river the missionaries and cannibals start out on.

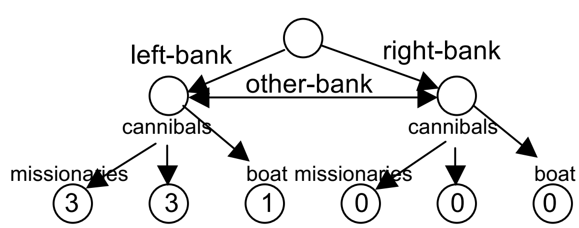
Here is one possible representation:

```
(state <s> ^right-bank-missionaries 0-3
           ^left-bank-missionaries 0-3
           ^right-bank-cannibals 0-3
           ^left-bank-cannibals 0-3
           ^right-bank-boat 0/1
           ^left-bank-boat 0/1)
```

Although this representation is adequate for solving the problem, it doesn't allow you to write general rules for proposing and applying operators. Using this representation, you would have to write separate proposal and application rules for when the boat is on each bank of the river. You would also have to write separate rules for moving cannibals and missionaries. When you have an attribute like right-bank-boat, the rules in Soar cannot match the different substructures, such as right-bank and boat. By representing each aspect separately using structured objects, you will find that it is possible to write very general operator proposal and application rules.

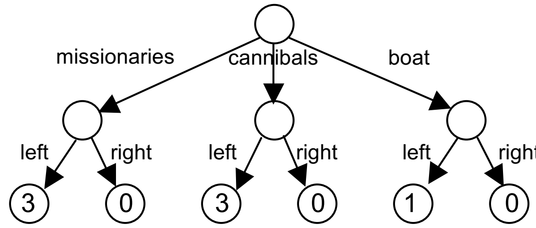
There are two obvious structured representations to choose from. One state representation has objects on the state for the two banks of the river, with subobjects for the missionaries, cannibals, and boat on that bank. To simplify later processing, an additional attribute (other-bank) can be added to the subobjects so that opposite bank can be matched easily. Below is a representation for the initial state:

```
(state <s> ^left-bank <l>
           ^right-bank <r>)
(<l> ^missionaries 3
     ^cannibals 3
     ^boat 1
     ^other-bank <r>)
(<r> ^missionaries 0
     ^cannibals 0
     ^boat 0
     ^other-bank <l>)
```



An alternative is to make the missionaries, cannibals, and boat the primary way of structuring the data, with the number of entities on each bank of the river as secondary.

```
(state <s> ^missionaries <m>
           ^cannibals <c>
           ^boat <b>)
(<m> ^left 3
     ^right 0)
(<c> ^left 3
     ^right 0)
(<b> ^left 1
     ^right 0)
```



For this problem, both of these representations are sufficient and they are similar in terms of the ease of writing the operators and goal tests. Soar programs for both are included as demonstration programs with the Soar release. For the remainder of this section, the first representation will be used because it is closer to the physical structure of the problem.

3. Initial State Creation: Initialize-mac

Just as in the Water Jug problem, you should create an initialization operator that names the state and creates all of the initial features on the state.

```
sp {mac*propose*initialize-mac
   (state <s> ^superstate nil
             -^name)
   -->
   (<s> ^operator <o> +)
   (<o> ^name initialize-mac)}

sp {mac*apply*initialize-mac
   (state <s> ^operator.name initialize-mac)
   -->
   (<s> ^name mac
        ^left-bank <l>
        ^right-bank <r>
        ^desired <d>)
   (<r> ^missionaries 0
        ^cannibals 0
        ^boat 0
        ^other-bank <l>)
   (<l> ^missionaries 3
        ^cannibals 3
        ^boat 1
        ^other-bank <r>)
   (<d> ^right-bank <dr>)
   (<dr> ^missionaries 3
         ^cannibals 3
         ^boat 1)}
```

4. Operator Proposal

The operators for this task move 1 to 2 individuals (missionaries or cannibals) across the river. In writing the proposal rules, it is easiest to break the operators into three classes:

- move one missionary or cannibal to the other bank
  - test that there is at least one of the given type on the bank with the boat
- move two missionaries or two cannibals
  - test that there is at least two of the given type on the bank with the boat
- move one missionary and one cannibal together
  - test that there is at least one of each type on the bank with the boat

Try to write an English description of the proposal for first operator.

```
mac*propose*move-mac-boat*1
If the name of the state is mac and there is one or more cannibal or
missionary on the same bank of the river as the boat, then propose moving
that cannibal or missionary across the river.
```

The other operator proposals are very similar:

```
mac*propose*move-mac-boat*2
If the name of the state is mac and there are two or more cannibals or
missionaries on the same bank of the river as the boat, then propose moving
two of that type across the river.

mac*propose*move-mac-boat*1
If the name of the state is mac and there is one or more cannibal and one or
more missionaries on the same bank of the river as the boat, then propose
moving one cannibal and one missionary across the river.
```

As in the Water Jug problem, you need to decide on a representation of the operator and its parameters. For this task, the operator parameters that make sense are:

- The name of the operator: move-mac-boat.
- The type of entities being moved: cannibal or missionary.
- The number of each type of entity being moved: 1 or 2.

The second two can be combined as a single attribute-value pair, with the type of entity being the attribute and the number being the value. This makes it easy to represent moving one or two entities of the same type as well as moving one missionary and one cannibal. To simplify later matching, you can also include the bank of the river that the boat is on. Also, for some of the reasoning, it will be useful to also represent how many different types of people are being moved, usually just one, but two when moving one missionaries and one cannibal. It is not necessary to include the bank of the river that the boat is on because it is represented in the current state.

The operator representation for moving one cannibal (with the boat) from the bank with object `l3` would be:

```
(<o> ^name move-mac-boat
     ^cannibal 1
     ^boat 1
     ^bank l3
     ^types 1)
```

Now try to write the first proposal as a Soar rule. To make it easier, a good approach is to initially write a very specific rule for one type of operator, and then attempt to generalize it by adding variables. To get started, you can try writing a proposal rule for just cannibals on the left bank of the river. That would be:

```
sp {mac*propose*move-mac-boat1
   (state <s> ^name mac
              ^left-bank <bank>)
   (<bank> ^cannibals > 0
           ^boat 1)
-->
   (<s> ^operator <o> + =)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^boat 1
        ^cannibals 1
        ^types 1)}
```

The operator is created with both an acceptable and a indifferent preference. In a later section you will explore adding search control.

You can generalize this rule by using a variable for the bank of the river, making it so that the proposal applies no matter which bank the boat is on. To be safe, this requires introducing a disjunctive (`<< left-bank right-bank >>`) test for the attribute.

```
sp {mac*propose*move-mac-boat*1
   (state <s> ^name mac
              ^<< right-bank left-bank >> <bank>)
   (<bank> ^cannibals > 0
           ^boat 1)
-->
   (<s> ^operator <o> + =)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^boat 1
        ^cannibals 1
        ^types 1)}
```

You can then further generalize the rule so that it can match against both cannibals and missionaries (but not the boat). This requires introducing a disjunctive (`<< cannibals missionaries >>`) test for the attribute of the bank object, and also a surrounding conjunctive test (`{ << cannibals missionaries >> <type> }`) to match the entity type to the variable `<type>`, which can then be used in the action. The final rule is:

```
sp {mac*propose*operator*move-mac-boat1
   (state <s> ^name mac
              ^<< right-bank left-bank >> <bank>)
   (<bank> ^{ << cannibals missionaries >> <type> } > 0
           ^boat 1)
   -->
   (<s> ^operator <o> + =)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^<type> 1
        ^boat 1
        ^types 1)}
```

Now try to write the second proposal that moves two missionaries or two cannibals as a Soar rule. This requires only minimal changes to the first. The only changes are to test for more than one missionary or cannibal, and to increase the number being moved to 2.

```
sp {mac*propose*operator*move-mac-boat2
   (state <s> ^name mac
              ^ << right-bank left-bank >> <bank>)
   (<bank> ^{ << cannibals missionaries >> <type> } > 1
           ^boat 1)
   -->
   (<s> ^operator <o> + =)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^<type> 2
        ^boat 1
        ^types 1)}
```

Now try to write the third proposal for moving one missionary and one cannibal.

```
sp {mac*propose*operator*move-mac-boat11
   (state <s> ^name mac
              ^ << right-bank left-bank >> <bank>)
   (<bank> ^missionaries > 0
           ^cannibals > 0
           ^boat 1)
   -->
   (<s> ^operator <o> + =)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^missionaries 1
        ^cannibals 1
        ^boat 1
        ^types 2)}
```

5. Operator Application

The operator application rules must change the state to reflect the movement of the boat and the missionaries and cannibals that cross the river. As part of applying the operators, it is not necessary to represent that the missionaries and cannibals are in the boat, only that they change banks of the river. The changes that need to be made to the state are to decrease the number of missionaries and cannibals that are moving from the bank of the river that the boat is on, and increase the number on the bank the boat is moving to. Similarly, the count of the boat (0 or 1) must be changed. You might try to come up with a set of rules to do this, but because of the operator representation, a single rule can make changes for moving cannibals, missionaries and the boat from either bank of the river to the other. The rule must test for an augmentation of the operator, such as `^boat`, `^cannibals`, or `^missionaries`, and then change the corresponding subobject on the state. The rule will fire in parallel for all entities being moved, including the boat.

Below is an English version of the required rule.

```
# mac*apply*move-mac-boat
# If there is a move-mac-boat operator selected for a type and number, then
# subtract the values of that type on the current bank and add those values
# to the other bank.
```

Converting this to a Soar rule is a bit tricky because of all of the variables. To simplify the conversion, we will start with a rule that applies the operator for moving one cannibal. Try to write this rule yourself.

```
# mac*apply*move-mac-boat*one*cannibal
# If there is a move-mac-boat operator selected for one cannibal, then
# subtract one from cannibal object on the left bank and add one to the
# cannibal object on the other bank.

      sp {apply*move-mac-boat*one*cannibal
         (state <s> ^operator <o>)
         (<o> ^name move-mac-boat
              ^cannibals 1
              ^bank <bank>)
         (<bank> ^cannibals <bank-num>
                 ^other-bank <obank>)
         (<obank> ^cannibals <obank-num>)
         -->
         (<bank> ^cannibals <bank-num> -
                            (- <bank-num> 1))
         (<obank> ^cannibals <obank-num> -
                             (+ <obank-num> 1))}
```

The above rule tests the operator to ensure that one cannibal is being moved `(<o> ^cannibals 1)` and to detect the bank of the operator. It then matches the number of cannibals on that bank, matching `<bank-num>`, via the other-bank attribute, matching `<obank-num>`. The actions of the rule modify the number of cannibals on the left bank by rejecting the current value `(^cannibals <bank-num> -`), and by asserting the new value which is the original value minus one (`^cannibals (- <bank-num> 1)`). Arithmetic operations such as addition, subtraction, and multiplication are done in Soar using prefix notation where the operation is given first followed by the operands.

One concern you might have about the above rule is that it will apply multiple times if there is more than one cannibal on the left bank of the river, moving each cannibal, one by one to the other bank. However, that will not happen because immediately after this rule fires (multiple times in parallel for each entity being moved) the rule that proposed the operator will no longer match, causing the operator to be removed. The operator proposal rule will no longer match because it tested the number of cannibals on the left bank of the river, which is changed by the rule. In addition, the boat will move from one bank to another by at the same time, providing a second reason for the proposal rule not to match. Thus, the operator will terminate immediately after the above rule fires.

The next step is to generalize this rule so it can apply to moving 1 or 2 cannibals. This requires replacing the test for ^cannibals 1 on the operator to ^cannibals <number> and then using <number> in the actions to subtract from the current

```
sp {apply*move-mac-boat*cannibal
   (state <s> ^operator <o>)
   (<o> ^name move-mac-boat
        ^cannibals <number>
        ^bank <bank>)
   (<bank> ^cannibals <bank-num>
           ^other-bank <obank>)
   (<obank> ^cannibals <obank-num>)
   -->
   (<bank> ^cannibals <bank-num> -
           (- <bank-num> <number>))
   (<obank> ^cannibals <obank-num> -
            (+ <obank-num> <number>))}
```

The final generalization is to replace the test for the ^cannibals attribute of the operator with a more general test that matches cannibals, missionaries, or the boat to a variable <type>. That variable is used to match the appropriate object on the state. This rule will now fire multiple times to move the boat as well as any cannibals or missionaries that are moving.

```
sp {apply*move-mac-boat
   (state <s> ^operator <o>)
   (<o> ^name move-mac-boat
        ^{ << missionaries cannibals boat >> <type> } <number>
        ^bank <bank>)
   (<bank> ^<type> <bank-num>
           ^other-bank <obank>)
   (<obank> ^<type> <obank-num>)
   -->
   (<bank> ^<type> <bank-num> -
                   (- <bank-num> <number>))
   (<obank> ^<type> <obank-num> -
                    (+ <obank-num> <number>))}
```

6. State and Operator Monitoring

Below are three rules that monitor the selected operator and the state (one rule for each bank that the boat is on).

```
sp {monitor*move-mac-boat
   (state <s> ^operator <o>)
   (<o> ^name move-mac-boat
        ^{ << cannibals missionaries >>  <type> } <number>)
   -->
   (write | Move | <number> | | <type>)}

sp {monitor*state*left
   (state <s> ^name mac
              ^left-bank <l>
              ^right-bank <r>)
   (<l> ^missionaries <ml>
        ^cannibals <cl>
        ^boat 1)
   (<r> ^missionaries <mr>
        ^cannibals <cr>
        ^boat 0)
   -->
   (write (crlf) | M: | <ml> |, C: | <cl> | B ~~~   |
                 | M: | <mr> |, C: | <cr> |  |)}

sp {monitor*state*right
   (state <s> ^name mac
              ^left-bank <l>
              ^right-bank <r>)
   (<l> ^missionaries <ml>
        ^cannibals <cl>
        ^boat 0)
   (<r> ^missionaries <mr>
        ^cannibals <cr>
        ^boat 1)
   -->
   (write (crlf) | M: | <ml> |, C: | <cl> |   ~~~ B |
                 | M: | <mr> |, C: | <cr> |  |)}
```

When you run your program, you will observe that your program runs forever and also sometimes visits states that are illegal according to the problem statement.

7. Desired State Recognition

The next step in creating a program to solve missionaries and cannibals is creating a rule that recognizes when a desired state has been achieved. Although a rule specific to the given problem can easily be written, it might be better to write one that is more general. For example, you might assume that the desired state will always have some number of missionaries, cannibals, on one bank of the river.
The action of the rule should be to print out a message that the problem has been solved and halt.

Write an English version of this rule.

```
# mac*detect*goal*achieved
# If the name of the state is mac and the number of missionaries and
# cannibals on one bank of the river in the desired state matches the number
# of missionaries and cannibals on the same bank in the current state, write
# that the problem has been solved and halt.
```

Translating this into Soar is relatively straightforward. Try to write your own before looking below.

```
      sp {mac*detect*state*success
         (state <s> ^desired <d>
                    ^<side> <ls>)
         (<ls> ^missionaries <m>
               ^cannibals <c>)
         (<d> ^{ << right-bank left-bank >> <side> } <dls>)
         (<dls> ^missionaries <m>
                ^cannibals <c>)
         -->
         (write (crlf) |The problem has been solved.|)
         (halt)}
```

If you run this with the earlier rules, the program should halt at some point; however, it is likely that it will visit a failure state and thus solved the problem incorrectly.

8. State Failure Detection

The next step is creating a rule that recognizes when a failure state has been encountered. According to the problem statement, a failure state is one where the cannibals out number the missionaries on one bank of the river. One condition that is often forgotten is to test that the number of missionaries is greater than zero. The action for this rule is to print out a message that the problem has failed to be solved, and then halt. Write an English version of this rule.

```
# mac*detect*goal*failure
# If the name of the state is mac and there are more cannibals than
# missionaries, and there is at least one missionary, on one bank of the
# river, then write that the problem has failed to be solved, and halt.
```

Translating this into Soar is relatively straightforward. Try to write your own before looking below.

```
sp {mac*evaluate*state*failure*more*cannibals
   (state <s> ^desired <d>
              ^<< right-bank left-bank >> <bank>)
   (<bank> ^missionaries { <n> > 0 }
           ^cannibals > <n>)
-->
   (write (crlf) |The problem has failed.|)
   (halt)}
```

Try running your complete program. Invariably the program will halt with failure because of the high likelihood of encountering a failure state.

9. Search Control: Undoing the Last Operator

In the current problem, when a failure is reached, the program halts. One possibility is to have the program start over again from the initial state. But if you were working on the problem, you would probably notice that you reached an illegal state, and you would go back one step by undoing the last operator and try to find another path. In order to undo the last operator, you must remember what it was. You can use some of the work you did on the Water Jug where you created a memory to *avoid* undoing the last operator to *prefer* to undo an operator when a failure state is achieved.

Given the representation of the move-mac-boat operator in working memory, you will have to write two rules to record the last operator, one that handles instances of the operator that move a single type of entity, and a second that handles instances of the operator that move one missionary and one cannibal. The action of these rules should create an augmentation of the state with information on the operator that is being applied. Try to write English versions of these rules.

```
mac*apply*move-mac-boat*record*last-operator*types*1
If an operator is selected to move one type of entity, then create an
augmentation of the state (last-operator) with the bank of the boat, the type
of entity being moved, the number, and that there is one type being moved.

mac*apply*move-mac-boat*record*last-operator*types*2
If an operator is selected to move two types of entity, then create an
augmentation of the state (last-operator) with the bank of the boat and that
there is two types being moved.
```

These can then be converted into Soar rules:

```
sp {mac*apply*move-mac-boat*record*last-operator*types*1
   (state <s> ^name mac
              ^operator <o>)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^{ << missionaries cannibals >> <type> } <n>
        ^types 1)
   -->
   (<s> ^last-operator <o1>)
   (<o1> ^types 1
         ^bank <bank>
         ^type <type>
         ^number <n>)}

sp {mac*apply*move-mac-boat*record*last-operator*types*2
   (state <s> ^name mac
              ^operator <o>)
   (<o> ^name move-mac-boat
        ^bank <bank>
        ^types 2)
   -->
   (<s> ^last-operator <o1>)
   (<o1> ^types 2
         ^bank <bank>)}
```

The rule to remove old records only has to test if the bank of the boat in the record of the last operator does not match the current bank that the boat is on because each time an operator is applied the boat changes banks.

```
mac*apply*move-mac-boat*remove*old*last-operator
```

If the move-mac-boat operator is selected and the bank in the last-operator is not equal to the bank of the current boat, remove the last-operator structure.

This can then be converted into Soar a rule:

```
sp {mac*apply*move-mac-boat*remove*old*last-operator
   (state <s> ^name mac
              ^operator <o>
              ^<lr-bank>.other-bank <o-bank>
              ^last-operator <lo>)
   (<lo> ^bank <o-bank>)
   -->
   (<s> ^last-operator <lo> -)}
```

Once you add these rules, you can now add rules that undo an operator whenever one leads to a failure state. However, you must first modify the rule that detects failure so that it doesn't halt the program, but just augments the state with failure:

```
# mac*detect*goal*failure
# If the name of the state is mac and there are more cannibals than
# missionaries, and there is at least one missionary, on one bank of the
# river, then augment the state with failure true.
```

Translating this into Soar is relatively straightforward.

```
sp {mac*evaluate*state*failure*more*cannibals
   (state <s> ^desired <d>
              ^<< right-bank left-bank >> <bank>)
   (<bank> ^missionaries { <n> > 0 }
           ^cannibals > <n>)
   -->
   (<s> ^failure <d>)}
```

Note that this rule only fires when there is an illegal state and it is not part of the application of an operator. Thus, it will retract and remove the failure augmentation automatically if the state changes and there is no longer an illegal state.

Now you can write rules that prefer operators that undo the last operator when there is failure. Just as before, this will require two rules, one for moving a single type of entity, and one that moves one missionary and one cannibal. Below is a general English version for both rules.

```
# mac*select*operator*prefer*inverse*failure
# If the name of the state is mac and the current state is a failure state
# and the last operator is the inverse of a proposed operator, then prefer
# that operator.
```

```
sp {mac*select*operator*prefer*inverse*failure*types*2
   (state <s> ^name mac
              ^operator <o> +
              ^failure <d>
              ^last-operator <lo>)
   (<o> ^name move-mac-boat
        ^<type> <number>
        ^types 1)
   (<lo> ^types 1
         ^type <type>
         ^number <number>)
   -->
   (<s> ^operator <o> >)}

sp {mac*select*operator*prefer*inverse*failure*types*1
   (state <s> ^name mac
              ^operator <o> +
              ^failure true
              ^last-operator.types 2)
   (<o> ^types 2)
   -->
   (<s> ^operator <o> >)}
```

After you have added these rules, your program will be able to solve the problem; however it will probably take a very indirect path to the solution. One reason is that after an operator has been successfully applied and generated a valid state, the inverse of that operator will often be selected, undoing the operator and wasting both operator applications. To avoid this, you can add two more rules that *avoid* undoing the last operator when the state is not a failure state.

```
# mac*select*operator*avoid*inverse*not*failure
# If the name of the state is mac and the current state is not a failure
# state and the last operator is the inverse of a proposed operator, then
# avoid that operator.

      sp {mac*select*operator*avoid*inverse*not*failure*1
         (state <s> ^name mac
                    ^operator <o> +
                   -^failure true
                    ^last-operator <lo>)
         (<o> ^types 1
              ^<type> <number>)
         (<lo> ^types 1
               ^type <type>
               ^number <number>)
         -->
         (<s> ^operator <o> < )}

      sp {mac*select*operator*avoid*inverse*not*failure*2
         (state <s> ^name mac
                    ^operator <o> +
                   -^failure true
                    ^last-operator <lo>)
         (<o> ^types 2)
         (<lo> ^types 2)
         -->
          (<s> ^operator <o> < )}
```

2M 3C 0B ~ 1M 0C 1B 1M 3C 0B ~ 2M 0C 1B 3M 2C 0B ~ 0M 1C 1B

After you have added these final rules, your program should solve the problem much quicker. However, you will notice that there is still some inefficiency. For example, at the initial state, it is possible for the program to attempt to apply the same operator to the state after it has failed with moving one or both the other missionaries. The figure below shows the initial state and the three successive states that it can cycle among. The problem is that only the most recent operator for a current state is recorded. It is not possible to associate all prior operators that applied to a state because the state is continually changing. In the next Part of the tutorial, you will learn how to use impasses and substates so that your programs can use look-ahead planning and solve this type of problem more directly.